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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2025

Third Semester

Faculty of Science

Branch I (A) – Mathematics

MT 03 C 12—FUNCTIONAL ANALYSIS

(2018 Admissions–Supplementary/2017, 2016 and 2015 Admissions–Mercy Chance)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Describe the span of $M = \{(1,1,1),(0,0,2) \text{ in } R^3\}$.
2. Define a Banach space. Give an example.
3. Determine the null space of the operator $T : R^3 \to R^2$ represented by $\begin{bmatrix} 1 & 3 & 2 \\ -2 & 1 & 0 \end{bmatrix}$.
4. Define an inner product space.
5. Give an example of an $x \in l^2$ such that we have strict inequality in Bessel's inequality.
6. Define a Sesquilinear form.
7. What are the adjoints of a zero operator O and an identity operator I.
8. Is l^1 reflexive. Justify your answer.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Give examples of subspaces of l^∞ and l^2 which are not closed.
10. Let X and Y be metric spaces and $T : X \to Y$ a continuous map. Show that the image of a compact subset M of under T is compact.

Turn over

11. Show that if a normed space X is finite dimensional, then every linear operator on X is bounded.

12. Show that the dual space of R^n is R^n.

13. Show that if Y is any closed subspace of a Hilbert space H, then $H = Y \oplus Z$, where $Z = Y^{\perp}$.

14. Show that every vector space $X \neq \{0\}$ has a Hamel basis.

15. Show that if a normed space X is reflexive then it is complete.

16. Show that if the dual space X' of a normed space X is separable, then X itself is separable,

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) State and prove Riesz's lemma.

    (b) Show that O if a normed space X has the property that the closed unit ball $M = \{x : \| x \| \leq 1\}$ is compact, then X is finite dimensional.

18. (a) Show that the dual space of l^p is l^q, where $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$.

    (b) Show that the dual of l' is l^{∞}.

19. (a) If a $x \perp y$ in an inner product space X, show that $\|x+y\|^2 = \|x\|^2 + \|y\|^2$.

    (b) State and prove $C[a,b]$ is not an inner product space.

20. (a) Show that if Y is a closed subspace of a Hilbert space H, then $Y = Y^{\perp\perp}$.

    (b) State and prove Bessel's inequality.

    (c) Explain Gram–Schmidt process.

21. (a) State and prove Riesz's theorem.

    (b) Define self adjoint, unitary and normal operators.

22. State and prove generalised Hahn–Banach theorem.

$(3 \times 5 = 15)$